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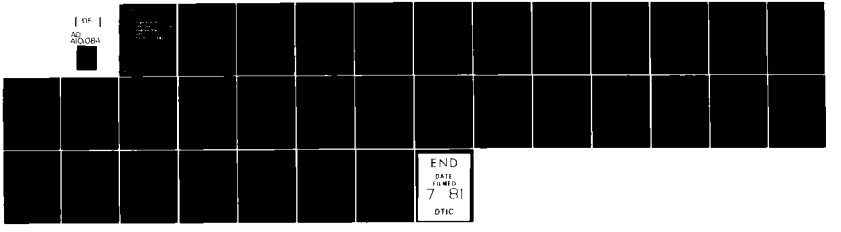

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20. Abstract

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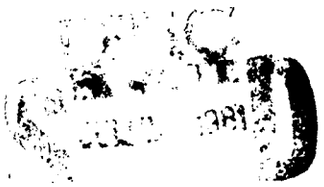

PSEUDO RANDOM STIMULUS RESPONSE OF COMBUSTION SYSTEMS

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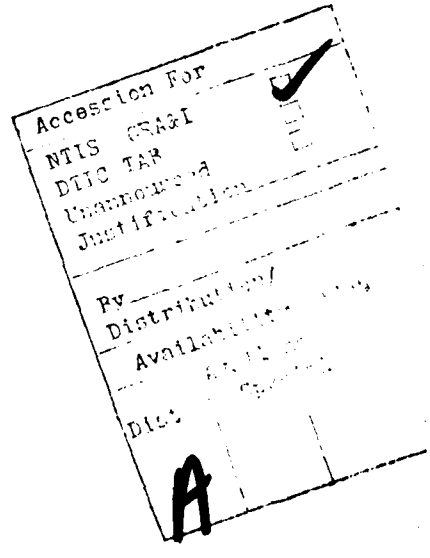
ABSTRACT

Combustor modelling methods involving stirred reactor networks require information on the volumes, inter-connections, mixedness and flowrates of the individual reactors. It is here proposed that the response of the system to a pseudo-random binary sequence (PRBS) stimulus can be used advantageously to fulfill this need. A survey is presented of the theory of the technique and method of interpretation of the results. The practicability of the method has been investigated experimentally using a water model of a ga turbine combuster; salt solution tracer was introduced into the primary flow in a PRBS and the conductivity at different points throughout the chamber was used to determine the impulse response and Z-transform. The results confirmed the anticipated sensitivity of the technique. Finally, the extension of the technique to the quantitative identification of non-linear elements is presented.



INTRODUCTION

The ultimate objective of combustion research is the optimization of practical combustor designs. Typical combustors (e.g. gas turbine combustors) involve the simultaneous interacting processes of three-dimensional turbulent flow, two-phase evaporating droplets, mixing, radiation and chemical kinetics. At the present time numerical prediction algorithms are becoming available[1,2] which can model all these processes to compute the hydrodynamic, thermodynamic and chemical quantities throughout a three-dimensional field. Complementary stirred reactor network algorithms permit the prediction of minor constituents (pollutants), again including such effects as droplet evaporation and unmixedness[2,3,4].

Current pressures to minimize pollutant formation, improve efficiency, tailor the pattern factor and widen the turndown ratio, together with the developing requirement to use more aromatic fuels, and even synthetic or coal derived fuels, suggest that the initial stages of combustor design would be greatly facilitated by reliable mathematical models based on fundamental principles.

In order to represent a gas turbine combustor, the present numerical finite difference algorithms, Fig.1. typically require about 3000 grid cells[3], whereas the reactor network models utilize about 7 reactors, Fig.2. The former model yields details of the spatial distribution but would require excessive computer time if complex kinetic equations were included. The latter model forfeits spatial resolution but permits complex chemical kinetic schemes to be incorporated. It is anticipated that the models can be eventually united by invoking the fact that optimum local grid size (for both models) is largely determined by the turbulence macro-scale. Thus spatial inhomogencities in the fuel feed are smoothed in a single stage stirred reactor having transverse dimensions of the order of the macro-scale. Similarly in the finite difference formulation, at least one grid point is required in a shear layer whose dimensions are again determined by the turbulence macro-scale.

On the other hand, the turbulence micro-scale determines the degree of mixing at the molecular level and hence the maximum possible completeness of combustion. In both models, the micro-mixing is often characterized by a turbulence dissipation parameter such as the eddy break-up parameter[5], or the τ_{SD} parameter[3]. Whilst the τ_{SD} parameter will easily permit the prediction of pollutant formation due to unmixed and hence unburned material by:

$$\eta_{c_1\ max} = 1/(1 + 1/\tau_{SD}) \approx (1 - 1/\tau_{SD}) \qquad (1)$$

it does not directly allow for non-linear effects in the chemical reactions. The significance of these effects depends on the amplitude of the fluctuations and the extent of the non-linearities. In the case of NO_x formation, it is well known that the kinetic equations have high activation energies and are therefore very non-linear. Previous efforts to account for this effect[6] have used the concentration variance g to compute the yield of NO_x, however the problem remains to predict g for a given combustor design.

A further problem arises if we wish to characterize a single reactor in a reactor network. Frequently, there is strong backmixing or re-cycle in the network and the boundaries of a single reactor are often drawn round the whole volume incorporating the backmixing. It is here proposed that the longitudinal extent of a single reactor is determined by either; the distance before the stream is divided (or joined by a second stream), or, the distance by which the turbulence decays to the level of negligible mixing. The transverse extent of each reactor is determined by the turbulence macro-scale as discussed above. Backmixing is thus defined by the flow rates and interconnections between the elementary reactors, and parameters such as blow-off only have meaning for that part of a network incorporating all recycle paths. Each elementary reactor therefore experiences a form of plug-flow with a specified degree of micro-mixing, and the mixing becomes zero in the conventional plug flow part of the combustor, thus limiting the combustion efficiency as indicated in Eq.1.

In order to approach this 'ideal' reactor network experimentally, powerful techniques are required and the purpose of this paper is to present some

appropriate procedures based on tracer response. The method is also applicable to the coalescence/dispersion (C/D) micromixing model[7].

In the C/D model, micromixing is simulated by considering the reacting mass to be made up of a large number of equally sized 'eddies' behaving as small batch reactors. Eddies undergo collision from time to time (at a mixing frequency β) and equalize concentrations when this happens. The dimensionless parameter $I_m = \beta/\alpha$, where $\alpha = 1/(\text{mean residence time } \tau_s)$, is analogous to τ_{SD} as shown in Fig.3. Computation proceeds by randomly selecting a pair of turbules within the reactor which average their properties at each interaction time interval (τ_s/I_mN), where N is the total number of turbules. Whenever a feedtime interval (τ_s/N) has elapsed, one turbule is selected at random to be removed from the reactor, and is replaced by fresh feed. However, as indicated by Pratt,[7] provided β is known, the C/D model can also be used to obtain both the mean and standard deviation of parameters such as temperature and concentration at the combustor exit, together with their various correlations. The model thus allows for non-linear chemical kinetic rates. In the case of the τ_{SD} model, the conventional treatment can be regarded as reducing the reactor volume to allow unmixed material to pass through unreacted, with the remainder of the reactor acting as a perfectly stirred system. This is reasonably true for gas turbine primary zones since τ_{SD} is so high. If the variation of the temperature and concentration about the mean could be related directly to τ_{SD}, then by analogy with a β/α model, it is possible to divide the reactor into 'cells' operating at appropriate conditions and thus allow for the non-linear chemical kinetic effects. The advantage of this approach compared to the C/D model is that it eliminates the computer generated random selection procedure and is therefore more efficient in the use of computer time.

Tracer stimulus response techniques have been previously applied to the study of chemical reactor design,[7] residence time distribution in combustors,[8,9] mixing,[10] turbulence,[11] etc. However, the present study has evolved largely from methods developed for process control and controller design in which stimulus response is used to identify transfer functions and fit stochastic models.[12-14]

In spite of the diversity of the fields and applications, the techniques and the relevant theories are basically the same and a good introduction to the subject is given by Godfrey.[15]

In the experiments, the system is disturbed by a suitable input signal, and the resulting system response together with the input signal are analysed by a variety of techniques to gain valuable information about the dynamics of the system.

Examples of forcing functions are: pulses, steps, sinusoids, random (noise), and pseudo-random binary sequences (PRBS). Although in principle the same information can be gained irrespective of the form of the input signal, the PRBS is a much more useful and powerful signal, and it is well established and widely used in the field of process control. On the other hand step and pulse functions are the dominant forms of input signals which have been used in combustion and reactor design studies, with little or no practical use of the PRBS.

As will be shown later, the advantages of the present approach are as follows:

a) the PRBS is on the average a steady state function and the combustor can therefore be operated at practically steady state.

b) the amplitude of pulse or step signals would have to be relatively large compared to the PRBS disturbance and this can result in system overloading and unquantified non-linearities.

c) for a given signal/noise ratio the experimental time is relatively modest.

d) many disturbance frequencies are generated for one easily generated code.

e) special methods described below can be used to identify the non-linear elements of the system.

f) quantitative estimates can be made of the turbulent mixing 'noise' contribution to the response.

The generation of a PRBS may be easily accomplished with a shift register connected as shown in Fig. 4. The number of stages, sequence period and feedback connections are shown in Table I. The sequence may be started with any non-zero

value in one or more of the registers, and it will be noted that m registers provide a sequence period of 2^m-1. A second class of PRBS sequences based on quadratic residue codes is described in Ref. 16.

THEORY (LINEAR SYSTEMS)

As illustrated in Fig.5, a linear system with input signal i(t) having an impulse response h(t) produces a response signal y(t) which together with the noise due for example to turbulence n(t) results in a measurable output signal Z(t) thus

$$Z(t) = y(t) + n(t) \tag{2}$$

The process response, y(t), is given by a weighted sum of inputs which have occurred in the past. Thus the past inputs are multiplied by the weighting function (or impulse response) h(t) as given by the convolution integral:

$$y(t) = \int_o^\infty h(t_1)\ i\ (t-t_1)dt \tag{3}$$

In practice, inputs occurring at times in the past greater than the settling time T_S have no effect on the present output so

$$y(t) = \int_o^{T_S} h(t_i)\ i\ (t-t_1)dt_1 \tag{4}$$

Thus from Equns 2 and 4.

$$Z(t) = \int_o^{T_S} h(t_1)\ i\ (t-t_1)dt_1 + n(t) \tag{5}$$

The cross-correlation function between the two signals i(t) and Z(t) is given by

$$R_{iz}(\tau) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} i(t)\ Z(t+\tau)dt \tag{6}$$

and similarly the auto-correlation function of a signal is:

$$R_{ii}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{o}^{T} i(t)\ i\ (t+\tau)dt \tag{7}$$

The auto-correlation thus represents the similarity of a signal with a time shifted version of itself. For example if a wide hand noise is compared with a time shifted verion of itself, only a small time shift is required to destroy the similarity, which does not recur. In the limiting case of white noise, the

the auto-correlogram is a Dirac delta function at zero phase shift. Pseudo-random binary signals have levels ± 1 (so that correlation only involves addition and subtraction) and may only switch level in a pre-determined sequence at time intervals Δt. The sequence repeats after N intervals (period $T = N\Delta t$) yielding an auto-correlelogram which is similar to a truly random signal as shown in Fig.6. It is this property which makes a PRBS a useful signal to study the system response since the relationship between the input-output cross correlation function and the input auto-correlation function[17] is

$$R_{iz}(\tau) = \int_0^{T_S} h(t_1)\, R_{ii}(\tau - t_1)\, dt_1 + R_{in}(\tau) \qquad (8)$$

where $R_{in}(\tau)$ represents the cross-correlation between the input and noise signal n(t). For a long correlation time (depending on signal/noise ratio as discussed below) and assuming that the noise signal is unaffected by the input signal, we obtain $R_{in}(\tau) = 0$. Now assuming R_{ii} could be represented by a Dirac delta function, Eqn. 8, would become the convolution of the impulse response with the Dirac function (for times less than $T = N\Delta t$). Now a useful property of the Dira delta function is that its convolution with the function yields the function, i.e.

$$\int_a^b \delta(t - t_o)\, f(t)\, dt = f(t_o) \quad \text{if } a < t_o < b$$
$$= 0 \quad \text{if a b interval does not contain } t_o. \qquad (9)$$

It therefore follows that the cross-correlation between input and output signals would approach in the impulse resnonse

$$R_{iz}(\tau) = K_1 h(\tau) \qquad (10)$$

where K_1 depends on the energy of the input pulse.

However, Fig.5. shows that for a PRBS with a period greater than the settling time (T_S) one auto-correlation triangle of height $(1 + \frac{1}{N})$ appears in the period together with a d c level of $-\frac{1}{N}$. Provided that Δt is small with respect to the time constants of the process, i.e. h(t) is approximately constant over the width of the auto-correlation triangle, the multiplication and integration

of Eq. 8. produces

$$R_{iz}(\tau) = (1 + \frac{1}{N})\ \Delta t\ h\ (\tau) - \frac{1}{N}\int_{o}^{T_S} h\ (t_1)\ dt_1 + R'_{in}\ (Z) \qquad (11)$$

where the second term is a d.c. level, with the integral $\int_{o}^{T_S} h(t_1)dt_1$ being the steady-state gain of the process. Over a long integration period the correlation $R'_{in}\ (Z) \to 0$. Thus the system dynamics can be conveniently studied with the aid of equation (11).

The results of PRBS experiments may be expressed either as the weighting sequence (impulse response) for discrete points

$$Z_t = \sum_{1}^{N_1} w_s i_{1-s} \quad \text{plus bias plus noise} \qquad (12)$$

or in terms of the Z transform (which is a closed form representation of the weighting sequence).

$$Z_t = \frac{w^{-1} \sum_{1}^{n} b_i\ w^{-j}}{1 + \sum_{1}^{n} a_i\ w^{-j}}\ i_t + \frac{\sum_{o}^{p} c_i\ w^{-j}}{1 + \sum_{o}^{p} d_i\ w^{-j}}\ n_t \qquad (13)$$

where w^{-1} is the shift operator defined as $w^{-1}y_t = y_{t-1}$ k represents the time delay in the system and n is the order of the system transfer function. The objective of the Z transform model analysis procedure is to evaluate the coefficients and order of the polynomials which give the best fit between the model predictions and the experimental data.

Amongst the generally available identification techniques including correlation methods, instrumental variables, maximum likelihood, etc. the generalized least square (GLS), algorithm[18] has proved to be statistically sufficient and is linear in its formulation. The algorithm can provide unbiased estimates, a desirable feature, as data required for the formulation of simple dynamic models is almost always corrupted by correlated noise and the need to evaluate unbiased estimates of the process model and of the noise becomes apparent. Additionally means exist to apply various diagnostic checks.

Equation 13 can be re-expressed (upon removal of any system integrations by data differencing) in the following convenient form

$$AZ_t = w^{-k} Bi_t + \frac{AD}{C} n_t \tag{14}$$

where A, B, C and D represent the polynomials in w^{-1}. Thus unless $ADC^{-1} \equiv 1$ the process parameter estimates using simple least squares will be biased. Hence the GLS aims to transform the term $ADC^{-1}n_t$ to an uncorrelated sequence n_t via the following steps:

i) The process parameters (a_i, b_i) are initially estimated using an ordinary least squares algorithm.

ii) The residuals, viz

$$e_t = \hat{A}Z_t - w^{-1} \hat{B}i_t \tag{15}$$

(where $\hat{}$ denotes estimated values) are analysed to be subsequently transformed by autoregression.

$$\hat{F}e_t = n_t$$

iii) The process input and output are filtered with the autoregression $\hat{F}$ to yield:

$$Z_t^F + \hat{F}Z_t \quad \text{and} \quad i_t^F = \hat{F}i_t \tag{16}$$

iv) i_t^F and Z^F are used to obtain a new least squares fit to commence another iteration cycle proceeding from step ii.

The final model is expressed in the form

$$\hat{A}(\hat{F}Z_t) = w^{-k} \hat{B} (\hat{F}i_t) + n_t \tag{17}$$

The validity of the estimated process and noise models given by the GLS algorithm have to be established by diagnostics checks for the statistical properties of the models. Currently available diagnostic algorithms utilize model order tests including determinant ratio test, F-ratio test, loss function analysis, pole-zero cancellation and tests for independence. In addition,

auto-correlations of the residuals and cross-correlations of the input and the residuals can be employed to reveal the adequacy of the fitted model.

Tentative values for the time delay and model order can be obtained via the impulse response evaluated directly by cross-correlation of the input and output for white inputs, Eqn. 10. An alternative means of extracting the same information is to use the determinant ratio test[14]. The test is especially valuable in that it limits the number of possible model orders and associated time delays prior to parameter estimation if the signal to noise ratio is reasonably high.

Loss function analysis and the associated F-ratio test are employed once the process and noise models have been derived by the GLS algorithm. They are primarily concerned with the behaviour of the error function in the neighbourhood of N_o taken to correspond to the actual system order. Additionally, residual manipulation in terms of auto-correlation functions provide a critical assessment of the structure and order of the estimated model. The auto-correlation function of the residuals is employed to yield an indication of the whiteness of the residuals. The cross-correlation function, on the other hand, establishes the statistical independence of the residuals and the process input.

The design of experiments and the interpretation of the results require particular consideration which may be illustrated by reference to the more familiar step test Fig.7. Indeed it is often convenient to carry out a step test to obtain criteria (T_d,T_p,T_s and noise variance) for the design of the PRBS experiment. The criteria to be fixed are the digit interval Δt, the period $N\Delta t$ and the signal amplitude $\pm\alpha$, determined as follows:

1. Clearly the PRBS time interval Δt must be short compared with the system time delay k if precision in the measurement of k is to be attained.
2. The time interval Δt should also be short compared to the estimated time constant T_p of the system (Fig.7.) although there is no advantage in excessivly short intervals and they would result in a decrease of the signal to noise

ratio of the output measurements.

3. The period N T should be about 1.25 to 1.5 times the settling time T_s. If the lag T_p is a first order time constant, then $T_s \sim 5T_p$. Longer periods may introduce problems due to signal drift although this can be reduced by trend analysis.

4. The amplitude α should be the largest which does not cause undue disturbance to the system. This follows since the first two terms on the right hand side of equation 11 increase as α^2 whereas $R'_{in}(Z)$ only increases linearly with α.

5. The sampling interval of the output measurement should preferably be synchronized to the start of PRBS steps. More than one sample per Δt interval will result in more data, which can be used either to digitally filter the input, thus improving the effective signal/noise ratio, or the resulting weighting sequence can be smoothed since it will consist of more points.

The static gain G of the system can be estimated from the sum of the weighting sequence ordinates or by the relation:

$$G = \sum_{1}^{n} b_i / (1 + \sum_{1}^{n} a_i) \qquad (18)$$

However, the signal/noise ratio is better if a simple step test is used to obtain G rather than a PRBS[19].

It is shown in Fig.8. from reference 20 that simple step response tracer results are insensitive to factors such as recirculation ratio unless very precise measurements are obtained in the vicinity of the roll-over point in the response curve. This region corresponds to the peak region of the impulse response curve, and it will be shown below that this is precisely the zone where PRBS methods give detailed results.

EXPERIMENT AND RESULTS

Due to our limited understanding of the reactor network and turbulence/kinetic interactions in gas turbine combustors, an experimental technique which can be applied in such complex three-dimensional flows is plainly required to study the phenomena. As a first step, the use of tracers in a water model has been

investigated. One important difference between water models (or air models) and hot combustion flows is that the former are isothermal. If geometrical similarity is maintained and Reynolds numbers are chosen to ensure that the model flow is turbulent, then as a result of the well known jet similarity characteristics, the flow patterns in the water model and prototype will be closely related. Using "partial modelling" (21) methods, in which the diameters of the holes in the combustor wall are increased in proportion to the square root of the cold jet to the hot chamber gas density ratio, the relative velocities and flow patterns are even more closely modelled. The model is not able to take full account of the effect of heat release on the turbulence structure, nor the effect of the turbulent fluctuations on the local heat release rate. Thus the micromixing 'noise' measurements will not be valid, however, deductions concerning the reactor network should be close approximations to the situation in a hot combustor. Some extensions of the techniques to direct measurements in hot combustion systems will be discussed later.

In the experiments a full scale water model of a research combustor of dimensions shown in Fig.9. was constructed. Salt solution could be injected at different points through a hypodermic needle, although in the results reported here the salt was always injected through one of the primary holes since we are particularly interested in the primary region of the combustor. The salt flow was modulated by a solenoid valve controlled by an LSI-11 computer, and the salt concentration was measured by a conductivity micro-probe which could be positioned accurately anywhere in the chamber. A detailed description of the experimental set up and measuring system is given in Ref. 22.

The investigation employed PRBS and step signals respectively as the system modes of excitation for parameter estimation purposes. The particular PRBS adopted used a bit interval Δt of 1·1 sec with a 15 bit period fed to the solenoid valve. A number of measuring stations, representative of the distinct mixing zones in the combustor were chosen (as shown in Fig.9.) for identification and parameter estimation.

A typical sequence of operational steps in the analysis is as follows: A determinant ratio test was used to limit the number of possible model orders for the range of time delays suggested by the deconvoluted impulse response. GLS parameter estimation was then applied for varying model orders and time delays. Typically noise model orders of 10-15 were adopted in an iterative scheme of 5-10 steps. Diagnostic checks comprising mainly the evaluation of the a c f of the residuals, the c c f between the input sequence and the residuals, and pole-zero cancellation were applied for varying model orders and time delays. Parameter estimation and subsequent validation were carried out in a strict iterative manner. Finally the impulse response corresponding to the estimated model was computed and this was compared with the original weighting sequence obtained from the cross-correlation of the input/output data.

The results obtained using step changes in salt concentration are illustrated in Fig. 10. The sharp cut-off at Station 2 within the primary jet is clearly demonstrated. The recirculation in the primary zone which behaves as a group of stirred reactors as defined above is clearly illustrated by the response at Stations 1, 7 and 8. The time constant of the exponential decay together with the known primary flow rate of 0.075 ℓ/s indicates an effective volume for this group of reactors of 0.2 ℓ which is consistant with the anticipated value. This clearly illustrates that turbulent exchange and recirculation involves virtually the whole volume of the primary zone, rather than one sixth which could be inferred by the cyclic symmetry. The series response of the secondary zone is indicated by the results at station 12. Correcting the measured time constant for the effect of the primary zone and the additional secondary flow gives the effective volume of the secondary stirred reactor as 0.15 ℓ.

Turning next to the PRBS technique, some of the Z transform results obtained are given as follows:

Station	Location	Order	Time Delay	a_1	a_2	b_1	b_2
7	Primary	1	2	-0.572	-	112.9	-
13	Primary	2	1	-0.753	-0.117	3.16	-116.5
9	Secondary	2	3	-0.293	-0.293	-78.3	-61.8
14	Dilution	2	4	-0.339	-0.528	-17.1	-112.3

The corresponding weighting sequences (impulse responses) are shown in Fig. 11. Integrating these curves gives a representation of the step response which can be used to determine the effective network volumes as discussed above. However, the integral tends to mask the detail which can be seen near the peak of the impulse curves as clearly illustrated by comparing the two cases at Station 14 (Fig. 11), thus it appears that the anticipated advantages of the PRBS method may be realized in practice.

The experimentally determined impulse response at Station 9 is particularly interesting since this point is on the opposite side of the axis to the trace injection. The response at this point is therefore due to turbulent exchange rather than convection, and the fact that it is almost identical to that above the axis confirms the results discussed above concerning the effective volume of the primary zone. The effect of this recycling reactor group size is to increase the throughput at blow-off, that is, enlarge the stability loop and altitude performance, as discussed in Ref. 23.

The relationship between the Z transform formulation and the more familiar (at least to combustion engineers) time constant concept is as follows (24).

1) For a simple first order system with no delay, the continuous system equation

$$(1 + TD)y(t) = Gi(t) \tag{19}$$

is, for a rectangular pulsed input, discretely co-incident with the discrete system satisfying

$$(1 - \delta w^{-1})y_t = G(1 - \delta)i_{t-1+} \tag{20}$$

where $\delta = e^{-1/T}$ or $T = (-\ln \delta)^{-1}$

(ii) For a delayed first order system, the equation

$$(1 + TD)y(t) = Gi(t-b-c) \tag{21}$$

corresponds to

$$(1 - \delta w^{-1})y_t = (\omega_o - \omega_1 w^{-1})i_{t-b-1+} \tag{22}$$

where $\omega_o = G(1 - \delta^{1-c})$ and $\omega_1 = G(\delta - \delta^{1-c})$

(iii) Similarly, a continuous second order system with delay obeying the equation

$$(1 + T_1D)(1 + T_2D)y(t) = i(t-b-c)$$

where T_1 and T_2 are real (rather than complex), is co-incident with the discrete system satisfying

$$(1-S_1w^{-1})(1-S_2w^{-1})y_t = (\omega_o - \omega_1w^{-1} - \omega_2w^{-2})i_{t-b-1+} \tag{23}$$

where $S_1 = e^{-1/T_1}$, $S_2 = e^{-1/T_2}$ and

$\omega_o = G(T_1 - T_2)^{-1} \{T_1(1-S_1^{1-c}) - T_2(1-S_2^{1-c})\}$

$\omega_1 = G(T_1 - T_2)^{-1} \{(S_1 + S_2)(T_1 - T_2) + T_2S_2^{1-c}(1 + S_1) - T_1S_1^{1-c}(1 + S_2)\}$

$\omega_2 = G\, S_1S_2(T_1 - T_2)^{-1} \{T_2(1 - S_2^{-c}) - T_1(1 - S_1^{-c})\}$

NB b is the number of whole increments Δt and c is the fraction, hence b + c = del

Thus using equation (22) the Z transform coefficients at Station 7 are equivalent to the continuous function Eqn. (21) with $\delta = 0.572$, c = 0, b = 2 gain = - 263, and the corresponding time constant $T = (- \ln \delta)^{-1} = 1.79$ secs.

It is apparent from the results at other Stations that the power of the PRBS to identify two or more time constants has been clearly demonstrated.

In principle, the value of the mixing parameter β(or τ_D) can be crudely obtained from the location of the maximum of the variance of the step response (see Ref. 25), however in well stirred systems such as the gas turbine combustor, the location of the maximum is so close to zero that it cannot be clearly distinguished. The equation relating the location of this maximum to β for a given residence time τ_S(= $1/\alpha$) is (25):

$$t_{1<< y(t) >>_{max}} = (1/(\beta - \alpha))\ \ln\ ((\beta + \alpha)/2d)$$

Fig. 12 shows the variance of a sequence of concentration steps applied to the water model and it can be seen that the maximum variance is located close to zero.

This is consistent with a high value of β as follows: Taking a typical residence time $\tau_S \approx 1$ sec, and $\beta > 100$ gives the time of the maximum variance $t/_{<<y(t)>>} < 0.04$ secs. Fortunately in highly stirred systems such as gas turbine combustors the results are insensitive to the exact value of τ_D(or β). In fact, the combustion efficiency is more significantly influenced by the uniformity of the fuel distribution, whereas the production of NO_x may be more influenced by the joint probability distribution of concentrations and temperature (which is at present unknown), than by the exact value of τ_D. Since probability distributions tend towards gaussian when they depend on the combination of several other probability distributions, almost whatever the shape of the separate distributions, it is likely that the effects of the p d f on pollutant formation can be represented by a set of about five parallel stirred reactor loops, the size and operating conditions of each weighted according to a portion of the gaussian curve.

If an accurate estimate of τ_D throughout the combustor is required it should be possible to extract this from the noise term in Eqn. (13). Although the auto-regressive model F used in the GLS algorithm, Eqn. (16), is often of high order, this is to ensure that $\hat{F} = (ADC^{-1})^{-1}$ provides an adequate representation of the rational transfer function $(ADC^{-1})^{-1}$. Estimates of the noise model coefficients d_i c_i can however be obtained by including an additional step in the GLS algorithm. This would consist of computing the least squares estimate of d_i, c_i in the model

$$\epsilon_t = \frac{D}{C} n_t \qquad (22)$$

where the residuals n_t are considered as input variables and the deterministic production errors $\epsilon_t = z_t - w^{-k} A^{-1} B i_t$ as output variables. Both ϵ_t and n_t are generated within the GLS algorithm and hence τ_D could be derived from the estimates associated with Eqn. (22).

Further developments of tracer techniques currently being investigated at Sheffield are based on hot combustion systems. In the first, a rapid response ($\tau \approx 0.1$ sec) infra red CO_2 analyser has been developed to study the overall chemical reaction dynamics in a coal fired fluidized bed combustor. The CO_2 analyser is used to monitor the conversion achieved in the bed in response to a PRBS input of coal or various reacting gases. In the second study, the flow reactor network in a hot, high intensity combustor is to be investigated using a mercury vapour tracer as described by Topps (26). He demonstrated that the vapour may be generated by passing brief electrical discharges through twin bore refractory tubing filled with mercury amalgam, and subsequently detected by the absorbtion of light generated by a hollow cathode mercury-neon lamp. In both studies, since only the a c response signals are required, long term stability is relatively unimportant. The experimental results using the methods detailed in this paper will be reported in d course.

At this point, the application of the PRBS stimulus technique to the measurement of the acoustic admittance (response function) of burners, rocket motors and propellants, boiler and furnace chambers, etc. should be mentioned. The particular advantage is, of course, associated with the large number of disturbance frequencies generated by the single code.

THEORY (NON-LINEAR SYSTEMS)

The representation of a mixing process followed by the non-linear Arrhenius rate equation in cascade with the instrumentation network is difficult to analyse experimentally because of the complexity of the system. However, recent results in non-linear identification theory[27,28] have shown that systems of this structure can be identified in terms of the individual component subsystems from measurements of the input and noise corrupted output signals only.

Classically identification techniques for non-linear systems are usually based on the Volterra series description:

$$z_1(t) = \int_{-\infty}^{\infty} g_1(\tau_1)\, i(t-\tau_1)d\tau_1 + \iint_{-\infty}^{\infty} g_2(\tau_1,\tau_2)\, i(t-\tau_1)\, i\,(t-\tau_2)d\tau_1\, d\tau_2$$

$$+ \ldots\ldots + \int_{-\infty}^{\infty} \ldots \int g_m(\tau_1, \ldots \tau_n)\, i\,(t-\tau_1) \ldots$$

$$\ldots\ldots i\,(t-\tau_m)d\tau_1 \ldots d\tau_m + n(t)$$

$$= \sum_{k=1}^{m} w_k(t) + n(t) \qquad (23)$$

or the related Wiener expansion[20], where the function $g_m(\tau_1, \tau_2 \ldots \tau_n)$ is termed the Volterra kernal of order m. The first term in the Volterra series is the well known convolution integral associated with linear systems and the higher order terms reflect the non-linearity of the process. Unfortunately, identification of the Volterra kernals often involves an excessive computational requirement and the physical interpretation of the final model in terms of the components of the original system is usually difficult to achieve. Consequently, very few authors have considered the identification of practical systems using these techniques.

However, recent results[27, 28] have shown that systems composed of linear dynamic and static non-linear elements can be identified using relatively simple extensions of well established linear techniques in a manner which preserves the system structure and provides estimates of the individual component subsystems.

In the present study only the cascade non-linear system illustrated in Fig.13. which has the structure of a stirred reactor network will be considered. The identification problem can be defined as identification of $h_1(t)$ the mixing dynamics, $F[\cdot]$ the Arrhenius rate equation and $h_2(t)$ the instrumentation network from measurements of $i_1(t)$ and $z_1(t)$.

If the input $i_i(t)$ is a separable process, and separability is preserved under linear transformation for $i_1(t)$, then the first degree cross-correlation

function is given by[27,28]

$$R_{i_1 i_1}(\varepsilon) = C_{FG} \iint h_2(\theta) h_1(\tau_1) R_{i_1 n}(\varepsilon - \theta - \tau_1)\, d\theta\, d\tau_1 + R_{i_1 n}(\varepsilon) \qquad (24)$$

Similarly, correlation between the input squared and the output yields the second degree correlation function, when separability is preserved under double non-linear transformation.[27,28].

$$R_{i_1^2 z_1}(\varepsilon) = C_{FFG} \iiint h_2(\theta) h_1(\tau_1) h_1(\tau_2) \overline{i_1(t-\theta-\tau_1)\, i_1(t-\theta-\tau_2)}$$

$$\overline{i_1^2(t-\varepsilon)}\, d\tau_1 d\tau_2 d\theta + R_{i_1^2 n}(\varepsilon) \qquad (25)$$

For the case when $i_1(t) = i(t) + b$ where $i(t)$ is a zero mean gaussian white process and b is a non zero mean level equations (24) and (25) reduce to

$$R_{i\, z_1'}(\varepsilon) = C_{FG} \int h_1(\tau_1) h_2(\varepsilon - \tau_1) d\tau_1 \qquad (26)$$

$$R_{i^2 z_1'}(\varepsilon) = C_{FFG} \int h_2(\tau_1) h_1^2(\varepsilon - \tau_1) d\tau_1 \qquad (27)$$

$$C_{FG} = \gamma_1 + 2\gamma_2 b \int h_1(\theta) d\theta + 3\gamma_3 \int h_1^2(\theta) + 3\gamma_3 b^2 \int h_1(\tau_1) h_1(\tau_2) d\tau_1 d\tau_2 + \ldots \qquad (28)$$

$$C_{FFG} = 2\gamma_2 + 6\gamma_2 b \int h_1(\theta)\, d\theta + \ldots \qquad (29)$$

where, provided $h_1(t)$ is stable bounded-inputs- bounded-outputs C_{FG} and C_{FFG} are constants, $F[\cdot] = \sum_{i=1}^{n} \gamma_i (\cdot)^i$, $R_{in}(\varepsilon)$, $R_i^2\, n(\varepsilon)$ tend to zero when $n(t)$ is independent of the input and the superscript ' is used to indicate a zero mean process.

The estimates of equations (26) and (27) are quite independent of the non-linear element $F[\cdot]$, the Arrhenius rate equation in this case, except for the constant scale factors C_{FG} and C_{FFG}. Correlation analysis thus effectively decouples the identification problem into two distinct steps; identification of the linear subsystems and characterisation of the non-linear element. Since we effectively have two equations in two unknowns, estimates of the individual linear subsystems $\mu_1 h_1(t)$, $\mu_2 h_2(t)$, where μ_1 and μ_2 are constants, can be obtained from equations (26) and (27) using a least squares decomposition

algorithm[27]. Once the linear subsystems have been identified the input and output of the non-linear element are effectively known. The problem is therefore reduced to fitting an appropriate function to the non-linear element by minimising the sum of squared errors.

The results of eqn. (26) and (27) also provide information regarding the system structure, or position of the non-linear element with respect to the linear subsystems. If the system is linear, $\gamma_k = 0$, $k \neq 1$ it can readily be shown that $R_{i^2 z_1'}(\varepsilon) = 0 \; \forall \; \varepsilon$ and this provides a very convenient measure of linearity. If the 1st. and 2nd. degree correlation functions are equal except for a constant of proportionality, then the system must have the structure of the Hammerstein model with $h_1(t) = \delta(t)$. However, if the 2nd. degree correlation function is the square of the 1st. degree correlation function, except for a constant scale factor, the system must have the structure of the Wiener model with $h_2(t) = \delta(t)$. Finally if none of the above conditions hold the system may have the structure of the general model but this must be confirmed by residual analysis or application of an algorithm by Douce[30].

Although the results presented above are relatively simple to compute, limitations of the input transducers may dictate that the input should be a pseudorandom sequence rather than a gaussian white noise process. Unfortunately, the results of eqns (26) and (27) do not hold for pseudorandom input sequences because of anomalies[31] associated with the higher order correlation functions of these inputs. However, by injecting a compound input defined as $i_1(t) = X_1(t) + X_2(t)$ analogous results can be obtained by isolating the correlation functions associated with the outputs $w_1(t)$ and $w_2(t)$ eqn. (23), of the first two Volterra kernels to yield[31].

$$R_{X_1' w_1'}(\varepsilon) = \beta_1 \gamma_1 \int h_1(\varepsilon-\theta)\, h_2(\theta)\, d\theta \tag{30}$$

$$R_{X_1' X_2' w_2'}(\varepsilon) = 2\beta_1 \beta_2 \gamma_2 \int h_1^2(\varepsilon-\theta)\, h_2(\theta) d\theta \tag{31}$$

where $X_1(t)$ and $X_2(t)$ are uncorrelated $R_{X_1 X_2}(\lambda) = 0 \; \forall \; \lambda$, zero mean pseudorandom

sequences with autocorrelation functions $R_{X_i X_i}(\lambda) = \beta_i \delta(\lambda), i = 1,2$. The disadvantage of this approach is the requirement to use multilevel compound inputs $K_i\ i_1(t)$ in order to isolate $R_{X_1' w_1'}(\varepsilon)$ and $R_{X_1' X_2' w_2'}(\varepsilon)$.
Since the results of equations (30) and (31) are dependent upon $X_1(t)$ and $X_2(t)$ having a zero mean value an obvious choice of input would be a compound ternary sequence. Pseudorandom binary sequences can however be used but the non-zero mean level of these sequences introduces a small bias into the estimate of eqn. (31) This bias tends to zero as the sequence lengths are increased and will be negligible in most applications.[31]

Thus by applying the above results to the stirred reactor network it should be possible to identify the mixing dynamics, parameters associated with the Arrhenius rate equation and the dynamics of the instrumentation network and this would undoubtedly provide a considerable insight into the operation of the process.

CONCLUSION

The problem of macro- and micro-mixing in combustors has been studied by PRPS stimulus tracer response techniques using a water model. The potential advantages of the method have been clearly demonstrated and future applications to the measurement of non-linear system parameters presented.

ACKNOWLEDGEMENT

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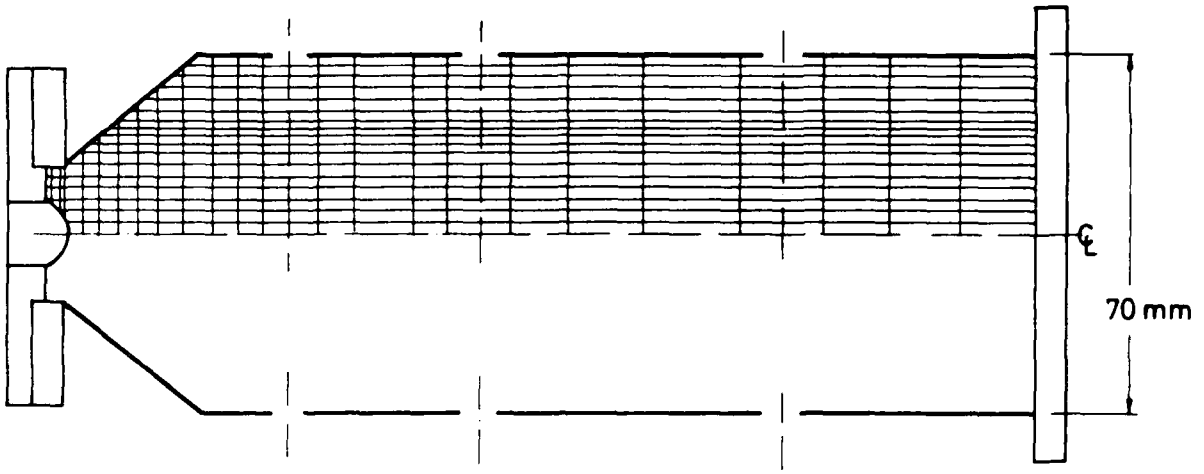
TABLE 1

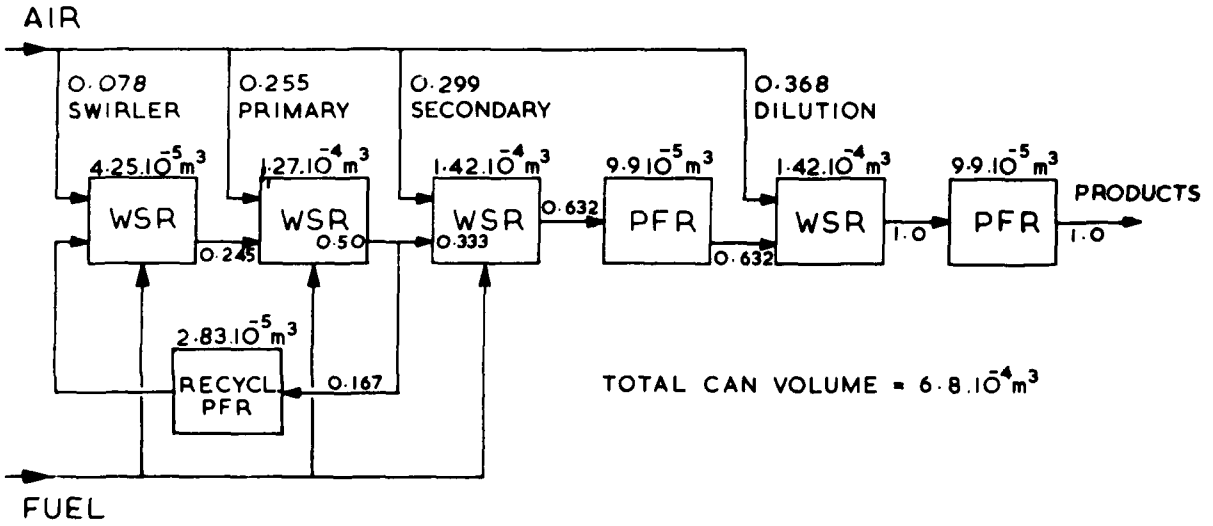
Number of shift register stages m	Period of sequence	Feedback to first stage modulo 2 sum of output of stages
2	3	1, 2
3	7	2, 3
4	15	3, 4
5	31	3, 5
6	63	5, 6
7	127	4, 7
8	255	2, 3, 4, 8
9	511	5, 9
10	1023	7, 10
11	2047	9, 11

N.B. The modulo 2 sum is defined as $1 \oplus 1 = 0 \oplus 0 = 0$
and $1 \oplus 0 = 0 \oplus 1 = 1$



LYCOMING COMBUSTOR, SCALE - FULL SIZE
GRID ARRANGEMENT 27 x 18 x 7 = 3 402 TOTAL POINTS

[illegible] grid.



[illegible]

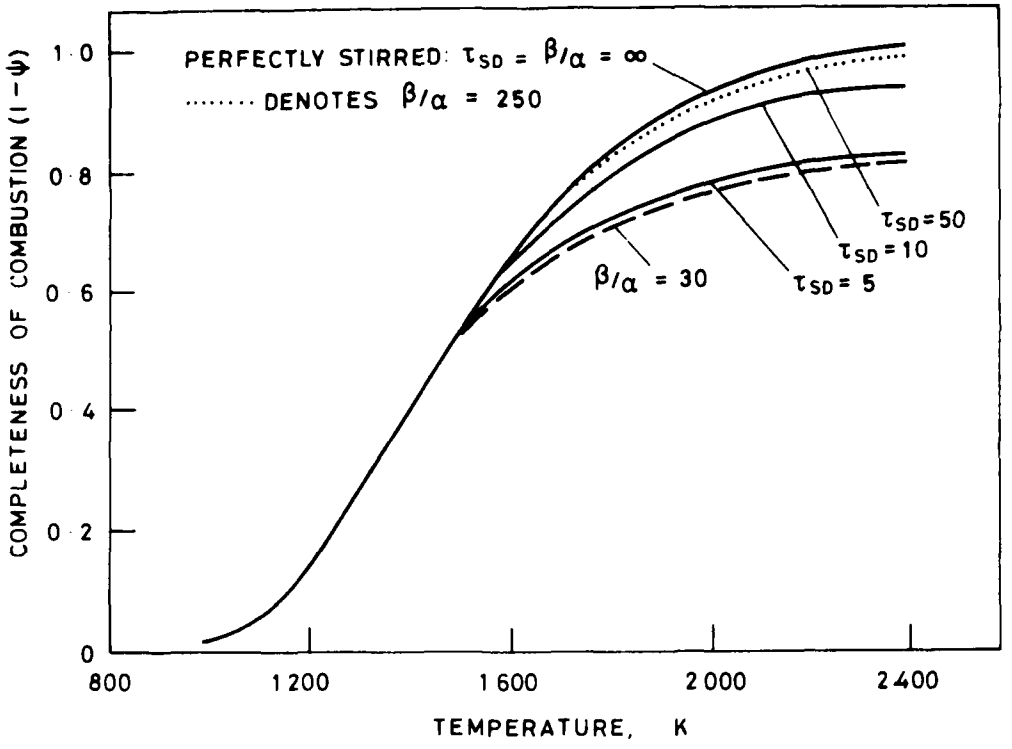


FIGURE 3. Comparison between τ_{SD} and β/α methods of computing stirred reactor performance.

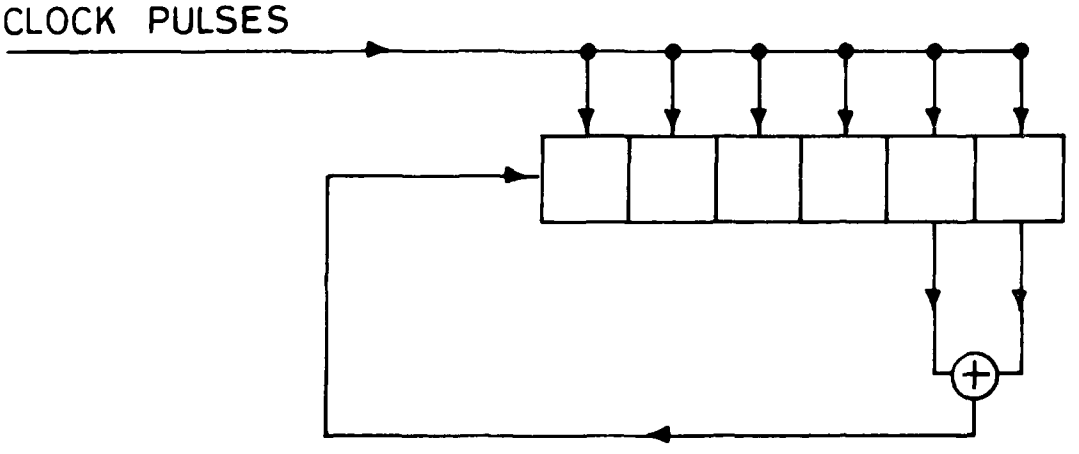


FIGURE 4. Shift register circuit for generation of a 63 digit sequence.

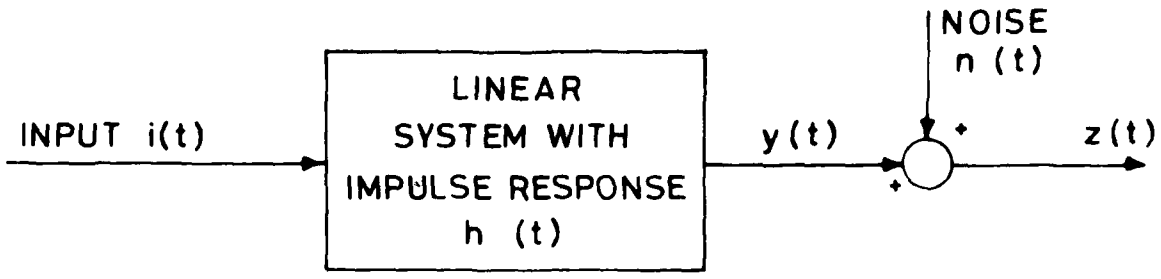


FIGURE 5. Block diagram of a simple system.

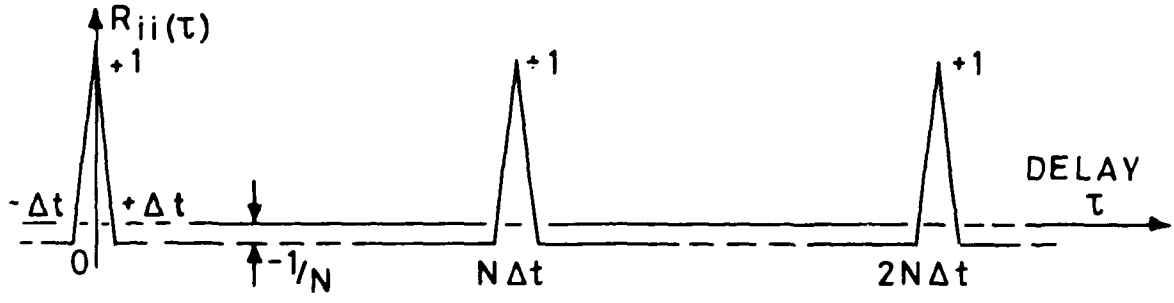


FIGURE 6. Auto-correlation function of a PRBS of period $N\Delta t$.

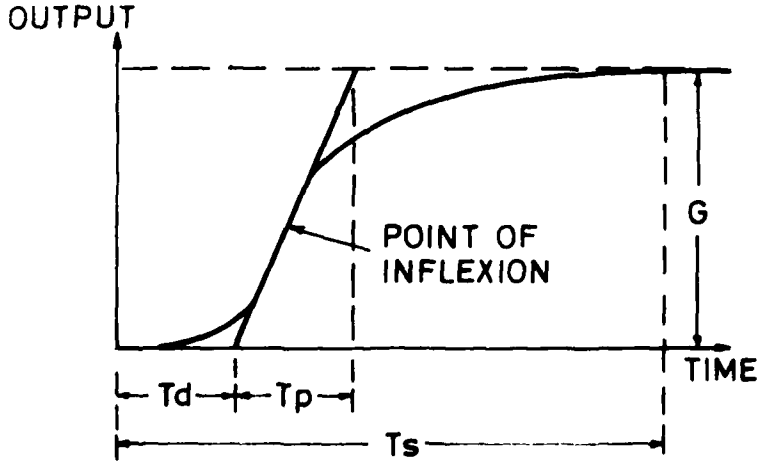


FIGURE 7. Typical response to a step input.

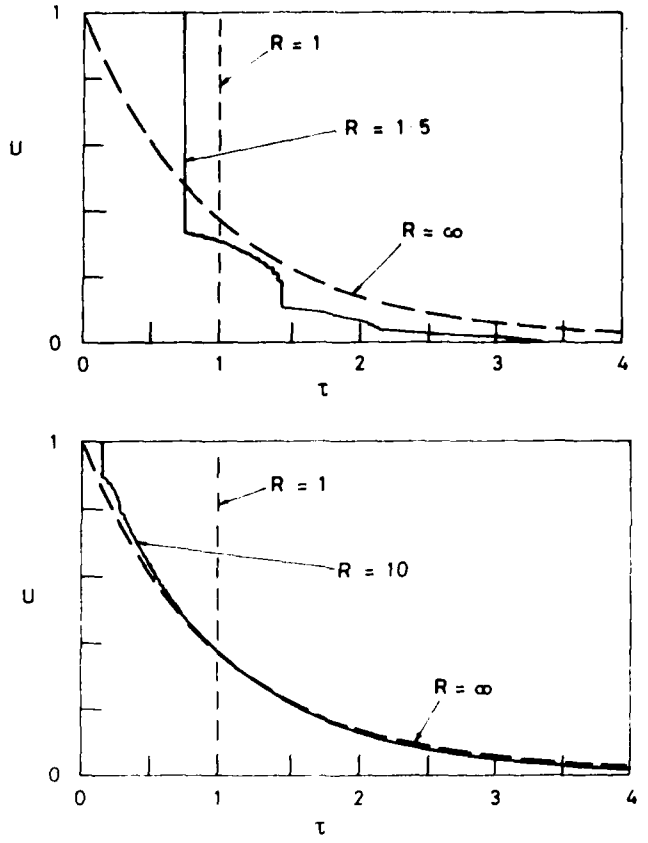


Fig. 8. Predicted tracer response functions for a jet stirred reactor for different recirculation ratios. The dashed curves correspond to a plug flow reactor (R=0) and a microbiked perfectly stirred reactor (R→). From Ref.

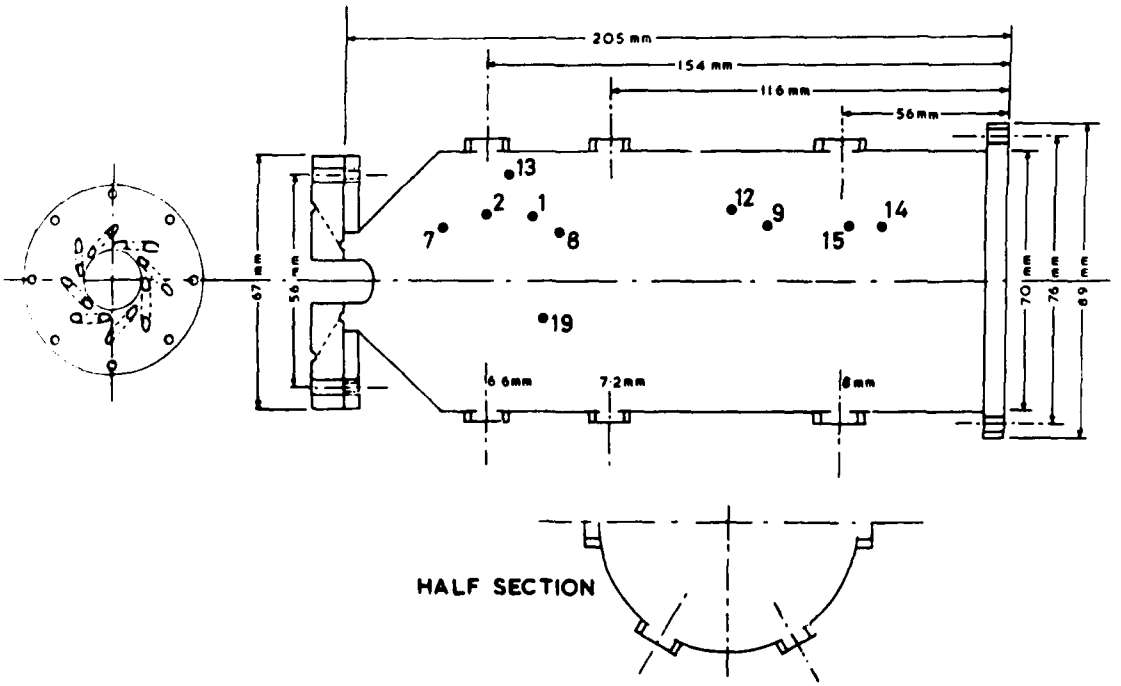


Fig. 9. [illegible] stirred reactor.

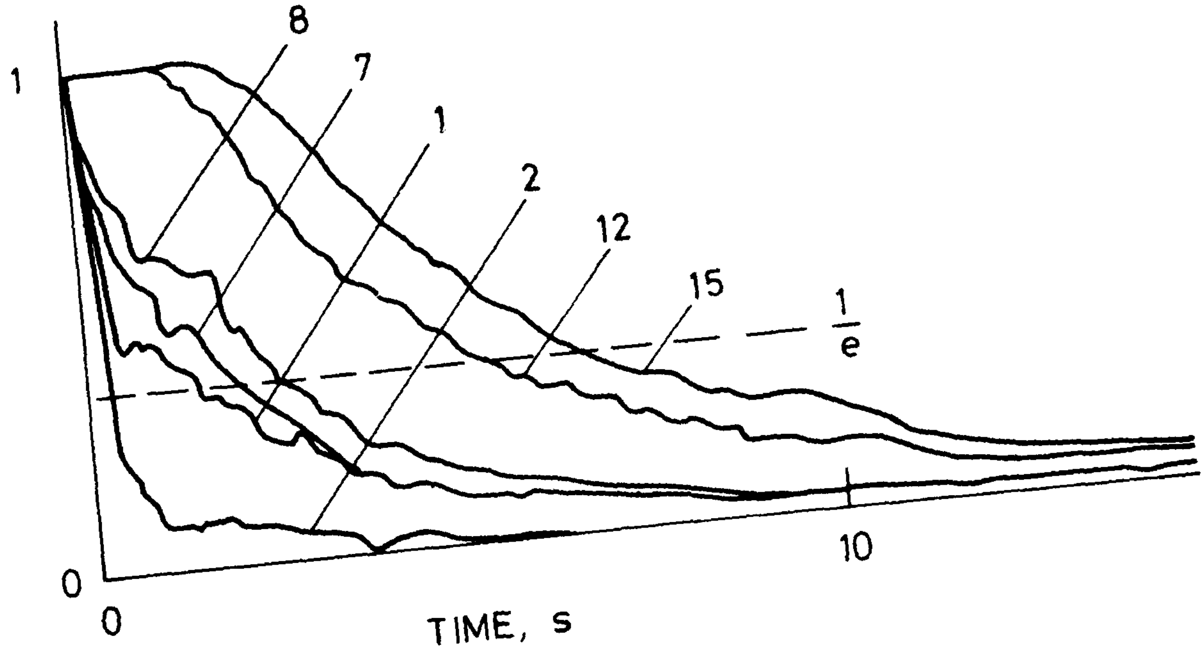
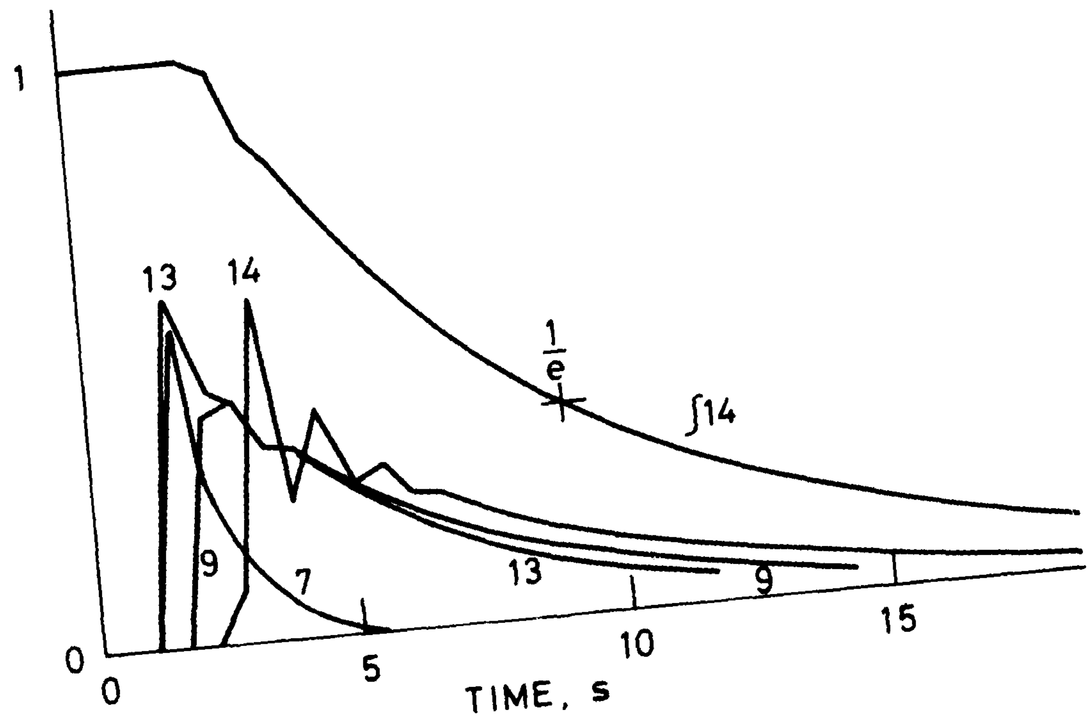


FIGURE 10. Variation of output concentration with time for a step input.



F[illegible]R[illegible] [illegible] response comp[illegible] for [illegible] input.

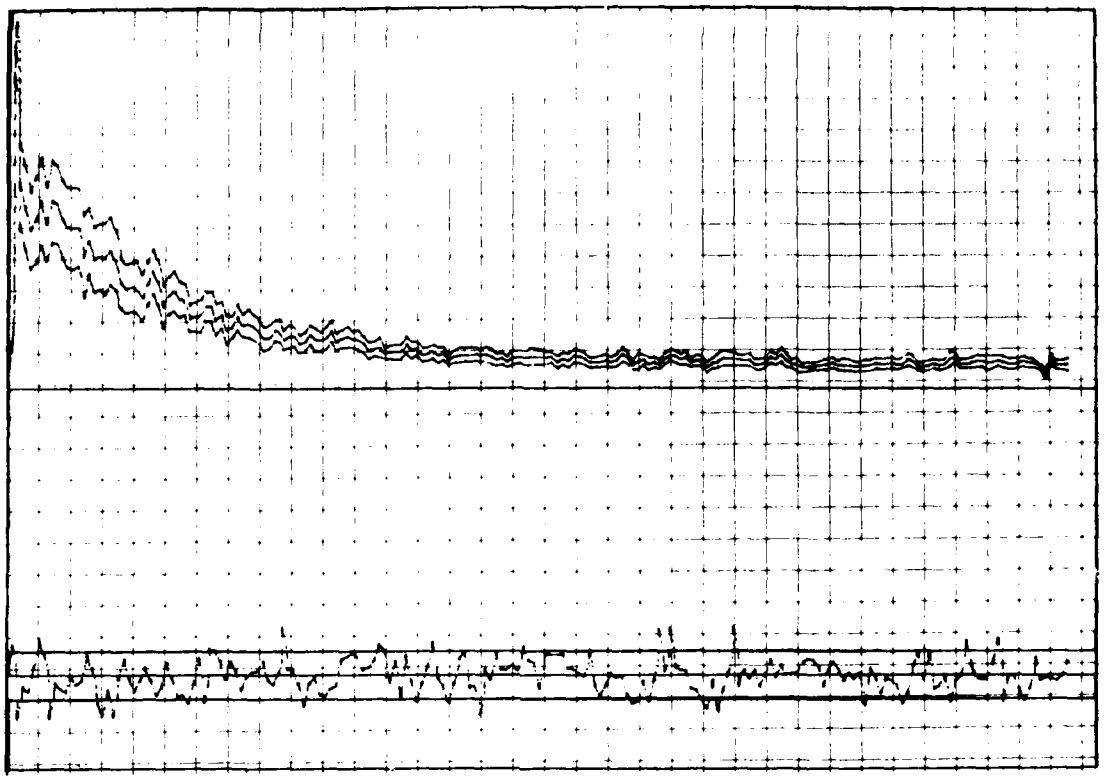


FIGURE 12. Step response output averaged waveform with confidence limits and trend.

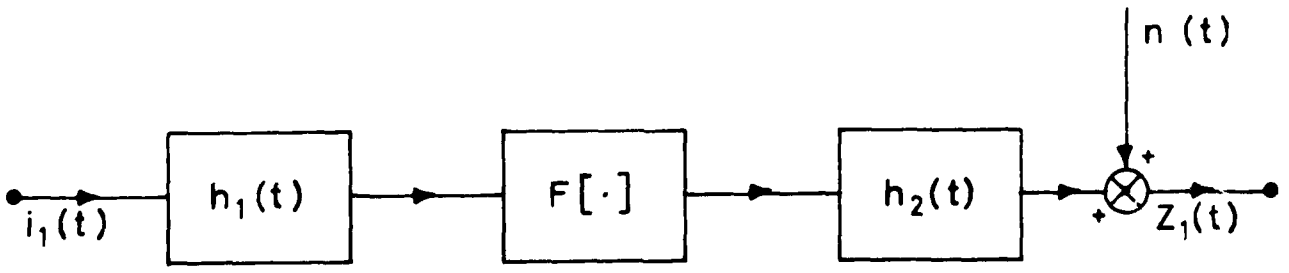


FIGURE 13. The non-linear model.

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